Differences in Differences

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These slides are part of the set of slides
A. Colin Cameron, Introduction to Causal Methods
https://cameron.econ.ucdavis.edu/causal/

November 2024

Introduction

- These slides give an introductory example of differences-in-differences (DID) estimation
  - DID is a method for causal inference
  - it is a general method for when an exogenous policy comes into being that effects one group more than another
  - it is often used with repeated cross-section data over time
  - but can also be used by comparing subgroups.
- DID relies crucially on an assumption called parallel trends
  - in the absence of treatment the trends for treated and untreated groups are equal.

- Separately the Stata file `dind.do` implements these methods
  - using dataset AED_HEALTHACCESS.DTA
- The data are from chapter 13.6 of A. Colin Cameron (2022) *Analysis of Economics Data: An Introduction to Econometrics* https://cameron.econ.ucdavis.edu/.
- Data are originally from Shinsuke Tanaka (2014), "Does Abolishing User Fees Lead to Improved Health Status? Evidence from Post-Apartheid South Africa", *American Economics Journal: Economic Policy*, 6(3), pages 282-312.

Outline

1. Introduction
2. Differences in differences: two time periods
3. Example: Access to health care and health outcomes
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Differences in differences: Two Time Periods

- Consider a "natural" experiment where an exogenous policy change (called a treatment) effects one group more than another.
- Let y denote the outcome and d denote the treatment
  - with $d = 1$ if treated and $d = 0$ if not treated.
- 1. Method 1: Treatment-control comparison (at a point in time)
  - Treatment effect = ($\bar{y}$ for treated) - ($\bar{y}$ for not treated)= $\bar{y}_{d=1} - \bar{y}_{d=0}$.
  - Problem: This is misleading if the treated and untreated groups differ in their characteristics
    - e.g. if the policy was targeted towards poor people.
- 2. Method 2: Before-after comparison over time for treated only
  - Treatment effect = ($\bar{y}$ for treated after treatment) - ($\bar{y}$ for treated before treatment)
  - Problem: Misleading if other things also effect the treated over time.
- 3. Differences-in-differences combines methods 1. and 2.
  - it uses change over time for the untreated to control for nontreatment changes over time (assuming both groups have the same time trend).

Differences in differences formula

- Introduce time before (pre) and after (post) the policy comes into effect
  - $t = 0$ is a time period before and $t = 1$ is a time period after.
- Then the difference in difference estimate of the effect of treatment is
  - DinD= $\Delta\bar{y}$ for those treated $-$ $\Delta\bar{y}$ for those not treated
  - $= (\bar{y}_{d=1,\text{post}} - \bar{y}_{d=1,\text{pre}}) - (\bar{y}_{d=0,\text{post}} - \bar{y}_{d=0,\text{pre}})$.
- Equivalently we can use
  - DinD= $(\bar{y}_{d=1,\text{post}} - \bar{y}_{d=0,\text{post}}) - (\bar{y}_{d=1,\text{pre}} - \bar{y}_{d=0,\text{pre}})$
  - the post-period difference in the two groups less that in the pre-period.
- DinD can be estimated by computing the four separate means and then computing the differences.

Regression computation

- The same difference-in-difference estimate can be obtained as the coefficient of $t \times d$ in the OLS regression

$$y_i = \beta_1 + \beta_2 t_i + \beta_3 d_i + \beta_4 t_i \times d_i + u_i.$$

  - where $t_i = 1$ in the post-period and $t_i = 0$ in the pre-period
  - and $d_i = 1$ if treated and $d_i = 0$ if not treated
  - $t_i \times d_i = 1$ if treated and in the post-period and $= 0$ otherwise.

- Proof: The model implies that y equals the following

	Treated ($d = 1$)	Not Treated ($d = 0$)	Difference over treatment
Pre ($t = 0$)	$\beta_1 + \beta_3$	β_1	β_3
Post ($t = 1$)	$\beta_1 + \beta_2 + \beta_3 + \beta_4$	$\beta_1 + \beta_2$	$\beta_3 + \beta_4$
Change over time	$\beta_2 + \beta_4$	β_2	**Diff in diff** $= \beta_4$!

Differences in differences regression computation

- So suppose we have data on each individual, not just the means.
- The OLS regression is

$$y_i = \beta_1 + \beta_2 t_i + \beta_3 d_i + \beta_4 t_i \times d_i + u_i.$$

- This is often written as

$$y_i = \beta_1 + \beta_2 Post_i + \beta_3 Treat_i + \beta_4 Post_i \times Treat_i + u_i.$$

- The difference-in-differences estimate is β_4.
- The advantages of using an OLS regression are
  - 1. A t-test of $H_0 : \beta_4 = 0$ is a test of statistical significance of the treatment
  - 2. We can add control variables as additional regressors.
  - 3. We can compute robust standard errors of $\widehat{\beta}_4$.

Example: Access to health care and health outcomes

- Does better access to health care lead to better health outcomes?
- Dataset AED_HEALTHACCESS has data on 1,071 South African children aged 1 to 4 years in 54 communities.
- In 1993 26 of 54 communities had access to a health care clinic.
- In 1998 all 54 communities had access to a health care clinic.
- Outcome y is `waz` is a weight-for-age $z-$score
- Treatment $d = 1$ if have access to a health care clinic.
- Time $t = 0$ in 1993 (pre-period) and $t = 1$ in 1998 (post-period).

Example (continued)

- Summary statistics for key variables

```
Variable      Storage   Display    Value
    name         type    format    label      Variable label
--------------------------------------------------------------------------------
waz             double   %6.2f                Weight for age z Score
hightreat       float    %9.0g                = 1 if community has clinic in 1993
post            float    %9.0g                = 1 if year==98 and =0 if year==93
postXhigh       float    %9.0g                = post times hightreat
waz             double   %6.2f                Weight for age z Score
whz             double   %6.2f                Weight for height z Score

. summarize waz hightreat post postXhigh waz whz

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
         waz |      1,071    -.205873    1.587432      -5.88       4.94
   hightreat |      1,071    .4276377    .4949671          0          1
        post |      1,071    .4668534    .4991332          0          1
   postXhigh |      1,071    .1979458    .3986373          0          1
         waz |      1,071    -.205873    1.587432      -5.88       4.94
-------------+---------------------------------------------------------
         whz |      1,071    .6390009    2.199942      -9.89       9.99
```

Results: Manual computation

- The following table gives the mean values of `waz`
  - for the high treated and low treated children
  - before and after the expansion in free health care.

	High treated		Low treated
Before (1993)	-0.545 ($n = 246$)		-0.414 ($n = 325$)
After (1998)	0.321 ($n = 212$)		-0.069 ($n = 288$)
Change over time	0.867		0.345
Difference in differences		0.521	

- High treated: `waz` increased by 0.867, from -0.545 to 0.321.
- Low treated: waz increased by 0.345, from -0.414 to -0.069.
- DID estimate is $0.867 - 0.345 = 0.521$.
- This is a very substantial effect
  - a third of a standard deviation change in `waz` for this sample.

Results: Regression computation

- Again greater access to health clinics increased waz by 0.521
- Since the treatment was at the community level, use cluster-robust standard errors with clustering on community
  - the standard error is 0.236 whereas heteroskedastic-robust s.e. is 0.194.

```
. * Diff-in-diff - no controls and cluster-robust standard errors
. reg waz postXhigh post hightreat, vce(cluster idcommunity) noheader
                           (Std. err. adjusted for 54 clusters in idcommunity)
```

waz	Coefficient	Robust std. err.	t	P>|t|	[95% conf.	interval]
postXhigh	.5216188	.2352991	2.22	0.031	.0496685	.993569
post	.3450874	.1371018	2.52	0.015	.070096	.6200788
hightreat	-.1310593	.1968084	-0.67	0.508	-.525807	.2636884
_cons	-.4141846	.1151423	-3.60	0.001	-.6451308	-.1832384

Further analysis

- A richer and better model
  - controls for community by adding fixed effects for each community
  - controls for each individual by adding regressors such as parental education and household income
- For child i in community c
  - $y_{ic} = \beta_1 + \beta_2 t_i + \beta_3 d_i + \beta_4 t_i \times d_i + \gamma_c + \beta_5 x_{ic} + \cdots + u_i.$

```
. * D in D with fixed effects for community and individual controls
. reg waz postXhigh post hightreat i.idcommunity      ///
>     fedu medu hhsizep lntotminc immuniz nonclinic,  ///
>     vce(cluster idcommunity) noheader
note: 242.idcommunity omitted because of collinearity.
                                (Std. err. adjusted for 54 clusters in idcommunity)
```

waz	Coefficient	Robust std. err.	t	P>|t|	[95% conf.	interval]
postXhigh	.6428807	.2710993	2.37	0.021	.0991243	1.186637
post	-.6807024	.3487963	-1.95	0.056	-1.380299	.0188944
hightreat	-.2911247	.2360665	-1.23	0.223	-.7646142	.1823648

Stata didregress command

- Stata `didregress` command defines the treatment variable to be $d_{it} = 1$ or $d_{it} = 0$
  - this is $d_i \times t_i$ in the previous notation
  - i.e. `postXhigh` in the current example (not `hightreat`)
- With group and time effects and control variables we give command

```
. didregress (waz fedu medu hhsizep lntotminc immuniz nonclinic) (postXhigh), ///
>      group(idcommunity) time(post)

Treatment and time information

Time variable: post
Control:       postXhigh = 0
Treatment:     postXhigh = 1
```

	Control	Treatment
Group		
idcommunity	29	25
Time		
Minimum	0	1
Maximum	0	1

Further Details

- We get the same ATET and standard error as the earlier `regress` command.

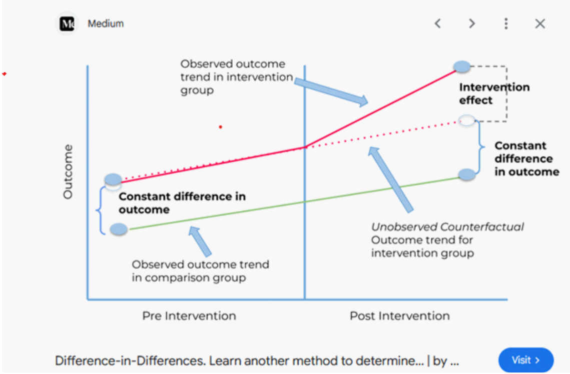
```
Difference-in-differences regression                 Number of obs = 1,071
Data type: Repeated cross-sectional

                         (Std. err. adjusted for 54 clusters in idcommunity)
------------------------------------------------------------------------------
             |               Robust
         waz | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
ATET         |
   postXhigh |
   (1 vs 0)  |   .6428807   .2710993     2.37   0.021     .0991243    1.186637
------------------------------------------------------------------------------
Note: ATET estimate adjusted for covariates, group effects, and time effects.
```

Further Details

- Differences-in-differences analysis is not restricted to one with time.
  - e.g. we might have a policy that affected only single women.
  - then compare the difference between married and single women with the difference between married and single men
    - assuming that without the policy the change from married to single would be the same for men and women.

Parallel Trends Assumption for Causality

- In order for differences in differences to have a causal interpretation we need to assume that the change over time in the outcome, in the absence of treatment, is the same for treated and untreated groups.



Parallel Trends Assumption (continued)

- Notation has three components
  - time: $t =$ pre or post
  - treatment: $d = 1$ if treated and $d = 0$ if not treated
  - **potential outcome**: $Y(1)$ if treated and $Y(0)$ if not treated
    - ★ for each person we can only observe one of $Y(1)$ or $Y(0)$.
- Define
  - $Y_{post}(1) =$ post-treatment Y if treated
  - $Y_{post}(0) =$ post-treatment Y if not treated.
- We want the average treatment effect on the treated
  $=$ expected outcome if treated $-$ expected outcome if not treated, for those who are treated
  $= E[Y_{post}(1)|d = 1] - E[Y_{post}(0)|d = 1]$
  - but $Y_{post}(0)|d = 1$ is not observed.

Parallel Trends Assumption (continued)

- The difference in difference estimate is
  - $\widehat{\gamma} = \Delta\bar{y}$ for the treated $-\Delta\bar{y}$ for the not treated
  - $\widehat{\gamma} = (\bar{y}_{\text{post},d=1} - \bar{y}_{\text{pre},d=1}) - (\bar{y}_{\text{post},d=0} - \bar{y}_{\text{pre},d=0})$
- This is an estimate of

$$\begin{aligned} \gamma = \quad & \{E[Y_{post}(1)|d=1] - E[Y_{pre}(0)|d=1]\} \\ & -\{E[Y_{post}(0)|d=0] - E[Y_{pre}(0)|d=0]\} \end{aligned}$$

  - since we observe $Y(0)$ in all cases except we observe $Y(1)$ for the treated in the post period.

Parallel Trends Assumption (continued)

- Add and subtract the unobserved $E[Y_{post}(0)|d=1]$

$$\begin{aligned}\gamma = \quad & \{E[Y_{post}(1)|d=1] - E[Y_{pre}(0)|d=1]\} \\ & -\{E[Y_{post}(0)|d=0] - E[Y_{pre}(0)|d=0]\} \\ & +E[Y_{post}(0)|d=1] - E[Y_{post}(0)|d=1].\end{aligned}$$

- Rearrange

$$\begin{aligned}\gamma = \quad & \{E[Y_{post}(1)|d=1] - E[Y_{post}(0)|d=1]\} \\ & +\{E[Y_{post}(0)|d=1] - E[Y_{pre}(0)|d=1]\} \\ & -\{E[Y_{post}(0)|d=0] - E[Y_{pre}(0)|d=0]\}\end{aligned}$$

- So $\gamma = \{E[Y_{post}(1)|d=1] - E[Y_{post}(0)|d=1]\}$ under the parallel trends assumption that

$$\begin{aligned}& \{E[Y_{post}(0)|d=1] - E[Y_{pre}(0)|d=1]\} \\ & = \{E[Y_{post}(0)|d=0] - E[Y_{pre}(0)|d=0]\}.\end{aligned}$$

Parallel Trends Assumption (continued)

- Note that if the parallel trends assumption holds in level, it will not hold in logs (and vice-versa).
- So we have to use the appropriate scaling of the outcome.

Differences in Differences: Multiple Time Periods

- Consider individual i in state s in year t, and the treatment of interest d_{st} occurs at the state-year level.
- Then we estimate the two-way fixed effects model
  - here ϕ_s and γ_t are state-specific and time-specific fixed effects.

$$y_{ist} = \phi_s + \gamma_t + \alpha d_{st} + \beta_1 x1_{ist} + \cdots + u_{ist}$$

- The key assumption is that of "parallel trends"
  - the time trend each period is the same for each state
    - ★ γ_t is the same for each state (rather than γ_{st})
  - this is partly testable in some applications using pretreatment data.
- Inference is based on standard errors clustered at the state (s) level
  - this leads to the "few clusters" problem if there are few clusters.
- OLS estimation is straightforward, but interpretation is difficult if treatment is staggered, occurring at different times for different states.
  - this is an area of current academic research.

Example: From Stata Documentation

- Example
  - y outcome is `satis` (Patient satisfaction score)
  - d treatment is `procedure = 1`
  - s group is hospital (there are 46)
  - t time is month (there are 7 months: January to July)
  - i is individual
- Treatment begins in April at 18 of the 46 hospitals

- Summary statistics

```
. summarize
```

Variable	Obs	Mean	Std. dev.	Min	Max
hospital	7,368	22.83822	13.57186	1	46
frequency	7,368	2.473398	1.163957	1	4
month	7,368	3.625	2.117778	1	7
procedure	7,368	.2079262	.4058512	0	1
satis	7,368	3.619074	1.05576	.5467862	9.712885

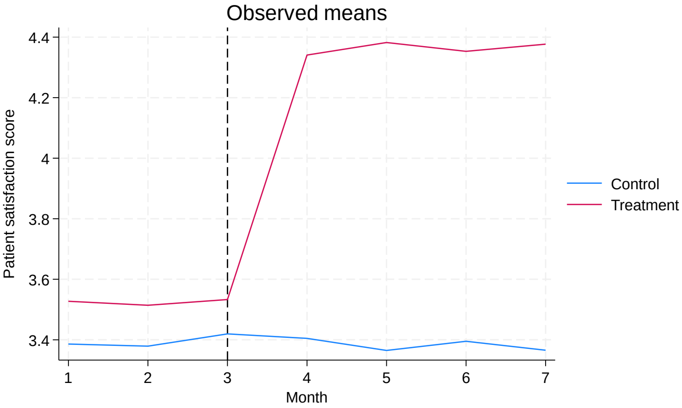
- Stata didregress command: Treatment effect is 0.84789

```
. didregress (satis)(procedure), group(hospital) time(month)

Treatment and time information

Time variable: month
Control:       procedure = 0
Treatment:     procedure = 1
-----------------------------------
             |   Control  Treatment
-------------+---------------------
Group        |
    hospital |        28         18
-------------+---------------------
Time         |
     Minimum |         1          4
     Maximum |         1          4
-----------------------------------

Difference-in-differences regression                  Number of obs = 7,368
Data type: Repeated cross-sectional

                               (Std. err. adjusted for 46 clusters in hospital)
-------------------------------------------------------------------------------
              |               Robust
        satis | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
--------------+----------------------------------------------------------------
ATET          |
    procedure |
 (New vs Old) |   .8479879   .0321121    26.41   0.000     .7833108     .912665
-------------------------------------------------------------------------------
Note: ATET estimate adjusted for group effects and time effects.
```

- Following gives the same estimate and st. error using regress

```
. * The following gives the same results as didregress
. regress satis procedure i.hospital i.month, vce(cluster hospital)

Linear regression                               Number of obs     =      7,368
                                                F(6, 45)          =          .
                                                Prob > F          =          .
                                                R-squared         =     0.5333
                                                Root MSE          =     .72384

                              (Std. err. adjusted for 46 clusters in hospital)
------------------------------------------------------------------------------
             |               Robust
       satis | Coefficient  std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
   procedure |   .8479879   .0321121    26.41   0.000     .7833108     .912665
             |
```

- Check if parallel trends in the pre-treatment period



- Visual test of parallel trends assumption
  - `estat trendplots`
- Formal test of parallel trends assumption
  - `estat ptrends`
- For more read the Stata pdf documentation.

References for DID

- These books are given in approximate order of increasing difficulty.
- A. Colin Cameron (2022), Analysis of Economics Data: An Introduction to Econometrics, chapter 13.6.
- Joshua D. Angrist and Jörn-Steffen Pischke (2015), Mastering Metrics, ch. 5.
- Cunningham, Scott (2021), Causal Inference: The MixTape, Yale UP, chapter 9.
- A. Colin Cameron and Pravin K. Trivedi (2022), Microeconometrics using Stata: Volumes 1 and 2, Second Edition, Stata Press, chapter 25.6.
- Joshua D. Angrist and Jörn-Steffen Pischke (2009), Mostly Harmless Econometrics: An Empiricist's Companion, Princeton University Press, chapter 5.
- A. Colin Cameron and Pravin K. Trivedi (2005), Microeconometrics: Methods and Applications, Cambridge University Press, chapter 22.6.
- Jeffrey M. Wooldridge, (2010), Econometric Analysis of Cross Section and Panel Data, Second Edition, MIT Press, chapter 6.5.

References for DID (continued)

- These books by non-economists are similar to *Mastering Metrics* in accessibility.
- Stephen L. Morgan and Christopher Winship (2015), Counterfactuals and Causal Inference: Methods and Principles for Social Research, Second edition, Cambridge University Press, chapter 11.3.
- Andrew Gelman, Jennifer Hill and Aki Vehtari (2022), Regression and Other Stories, Cambridge University Press, especially chapters 21.4.
- These are current more advanced econometrics articles
- B. Callaway and P.H.C. Sant'Anna (2021), "Difference-in-Differences with multiple time periods," Journal of Econometrics, 225, pages 200–230.
- Jeffrey M. Wooldridge (2021), "Two-way fixed effects, the two-way Mundlak regression, and difference-in-differences estimators," http://doi.org/10.2139/ssrn.3906345.